# Vertical Ascent from Earth to Geosynchronous Orbit

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The concept of ascending vertically from the equator on Earth to the point in geosynchronous orbit directly above was investigated. The gravity losses were found to be so great that vertical ascent is not practical with only chemical-rocket propulsion. With laser propulsion, propellant requirements with vertical ascent are reduced and might allow the use of single-stage vehicles from Earth to geosynchronous orbit. Combined or composite chemical and laser propulsion is shown to be useful. A gravity ladder suspended from geosynchronous orbit part of the distance to Earth is shown to reduce the vehicle propulsion requirements.

#### Nomenclature

A = acceleration in multiples of Earth's surface gravity

 $a_L$  = acceleration initially from laser propulsion

 $a_m$  = acceleration maximum

 $a_0$  = acceleration at liftoff

A =area of cross-section of gravity ladder

 $A_{\theta}$  = area at bottom of gravity ladder

 $F_F$  = body force toward Earth  $F_S$  = body force toward space

G = gravitational constant

h = altitude

I = integral for mass of gravity ladder

 $I_{SP,2}$  = specific impulse of laser

 $I_{SP}$  = combined or composite specific impulse m = mass of Earth arm of gravity ladder  $m_C$  = chemical propellant mass flow

 $\dot{m}_L$  = laser propellant mass flow

M =mass of Earth

 $P_c$  = power of chemical engine  $P_L$  = power of laser engine

 $P_t$  = total power

R' = ratio of initial mass of vehicle to final mass on orbit

r = radius from center of the Earth

 $r_G$  = radius to GEO

 $r_0$  = radius to bottom of gravity ladder

t = tension in gravity ladder

T = thrust

 $\theta$ 

 $T_I$  = laser temperature  $T_I$  = total temperature  $\Delta V$  = velocity increment

 $\Delta V_I$  = velocity increment with chemical-rocket propulsion

w =mass at end of combined burn divided by mass at

beginning of combined burn = thrust angle above horizonal

 $\sigma$  = stress in gravity ladder

 $\rho$  = density of gravity ladder

 $\omega$  = rotational rate of Earth

#### Introduction

UCH of today's space transport business involves placing satellites in geosynchronous equatorial orbit (GEO). The state-of-the-art approach is to use the Space Shuttle for the first part of the trip, from Earth to low Earth orbit (LEO). An orbit transfer vehicle (OTV) then propels the satellite to GEO. Advanced vehicles have been studied for

the next generation of Earth-to-GEO transportation, such as a fully reusable Earth-to-orbit vehicle and a space-based OTV.

In the more distant future, as traffic increases and technology improves, alternate approaches may prove beneficial for Earth-to-GEO transportation. One alternate approach, vertical ascent, will be evaluated in this paper. The primary reason for considering vertical ascent is that two variations from chemical-rocket propulsion—laser propulsion and a gravity ladder—are especially compatible with vertical ascent. These variations will also be discussed.

#### Vertical Ascent Concept

The vertical ascent concept is illustrated in Fig. 1 as seen from above the North Pole. The dashed lines connect the launch site on the equator and the destination in GEO, directly above the launch site, at launch and at various times during ascent. The vehicle ascends vertically, as viewed from the launch site, and at all times is along a dashed line between launch site and the destination. As seen from inertial space, the vehicle rises rapidly, initially covering two-thirds of the altitude change in about 2 h then gradually approaches GEO over the next 12 h for the example shown. As the vehicle rises, its tangential velocity must increase from that at the Earth's surface, 465 m/s, to that at GEO, 3080 m/s, to keep the angular velocity constant at one revolution per day. Since the vehicle is not required to achieve any intermediate orbits, the vertical velocity increases from zero at the Earth's surface, reaches some maximum during the flight, and returns to zero at GEO. The maximum velocity depends on the acceleration profile. For the example shown, the initial acceleration  $a_{\theta}$  (in multiples of the acceleration of gravity at the Earth's surface), is 1.3, and the maximum acceleration  $a_m$  is 3.0. The resulting maximum velocity is 8430 m/s. The velocity does not exceed

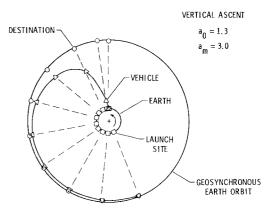


Fig. 1 Illustration of vertical ascent.

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1800 m/s in the atmosphere; therefore, ascent heating is not a concern.

# **Chemical-Rocket Propulsion**

The thrust history for a typical vertical ascent trajectory with chemical rocket propulsion is shown in Fig. 2. The thrust scale shown is appropriate for a vehicle gross mass of 1.5 Gg (millions of kilograms). As the vehicle rises, the thrust increases as the atmospheric backpressure falls. As the vehicle mass is reduced by propellant consumption, the acceleration reaches the selected maximum value, which is 3.0 for the case shown. After this point, the thrust is reduced continously to keep the acceleration at the limit.

The thrust angle  $\theta$  is vertical initially and decreases slightly as the vehicle rises. The eastward thrust component accelerates the vehicle just enough to keep it above the launch site. The eastward velocity must increase with altitude to maintain the same angular velocity as the radius increases. At some point in the ascent, the vertical thrust component is no longer needed because the velocity is sufficient to carry the vehicle to GEO with only the eastward thrust component. For the case shown, this point is at an altitude of 162 km. Above this altitude,  $\theta = 0$  deg. Only a small eastward thrust is required, and the thrust level drops at this point.

The chemical-rocket propulsion data used represent a Space Shuttle Main Engine (SSME) modified with a two-position nozzle. The trajectories were calculated using the program to optimize simulated trajectories (POST).<sup>2</sup> For the vertical ascent with  $a_0 = 1.3$  and  $a_m = 3.0$ , the total ideal velocity increment ( $\Delta V$ ) is 17.74 km/s.

The conventional approach to Earth-to-GEO transportation, the Hohmann ascent, is to insert the vehicle into the perigee of an elliptical orbit which has an apogee at GEO altitude. After a coast nearly to apogee, the velocity must be increased to circularize the orbit. In order to provide a basis

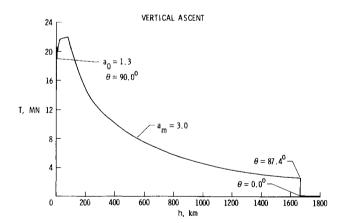


Fig. 2 Chemical propulsion thrust history.

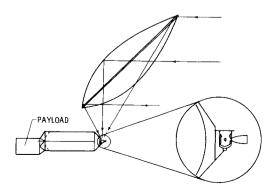


Fig. 3 Schematic of a laser-propelled vehicle.

for comparison with the vertical ascent, Hohmann ascent trajectories were calculated using the same propulsion characteristics. The insertion perigee was held constant at 92.6 km. The total velocity increment results for several combinations of  $a_0$  and  $a_m$  for both vertical ascent and Hohmann ascent were given in Ref. 3. The results indicate that the vertical ascent requires a significantly greater  $\Delta V$  than the Hohmann ascent, about 18 km/s rather than 13 km/s.

The difference in  $\Delta V$  between Hohmann ascent and vertical ascent is mostly due to gravity losses. Hohmann ascent benefits from centrifugal acceleration by achieving a velocity greater than circular-orbit velocity before insertion. The velocity remains greater than circular-orbit velocity for most of the coast period. Vertical ascent, on the other hand, does not achieve circular-orbit velocity before reaching GEO, and the majority of the thrusting is directly opposed to gravity. Because of the increase in  $\Delta V$  requirements, vertical ascent is impractical for vehicles with only chemical-rocket propulsion. Alternatives to chemical-rocket propulsion were explored and are discussed below.

#### Laser Propulsion

Laser propulsion has been the subject of several studies and some experimentation4 and offers the potential of specific impulse values up to 2000 s with hydrogen propellant. Coupling the laser beam power to the propellant can be accomplished in a variety of ways.4 Space limitations prohibit discussing details of laser propulsion herein. Figure 3 is a schematic of a typical laser-propelled vehicle. The inert mass of a laser propulsion system on a space transportation vehicle is greater than that for chemical-rocket propulsion but less than that for many advanced propulsion systems such as solar-electric propulsion. Unfortunately, laser propulsion is not very compatible with Hohmann ascent trajectories because of the line-of-sight requirement. If it is used for Earth-to-orbit vehicles, laser propulsion must provide very high thrust levels and operate through the atmosphere. For orbit transfer, the vehicle does not stay near the laser source all the time thrust is needed.

Laser propulsion, in combination with chemical-rocket propulsion, is quite compatible with vertical ascent. Chemical-rocket propulsion provides high thrust for liftoff and the initial acceleration through the atmosphere. Laser propulsion provides lower thrust in a vacuum to complete the ascent. The laser source is located at the destination in GEO, and the vehicle flies straight up the laser beam (Fig. 4). The laser propulsion can provide a larger portion of the total impulse than for a conventional Earth-to-orbit vehicle and a laser OTV. Locating the laser at the launch site would also be possible if the atmospheric losses are not excessive.

If laser propulsion is used for part of the ascent, the performance may be improved because of the higher specific impulse of laser propulsion. One way to measure the per-

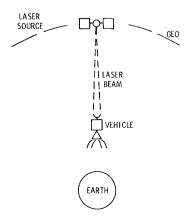


Fig. 4 Illustration of laser propulsion.

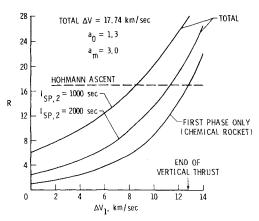


Fig. 5 Effect of laser propulsion during a portion of ascent on mass ratio. Total  $\Delta V$  of 17.4 km/s is based on the thrust history of Fig. 2.

formance is to determine the ratio of the initial mass to the final mass, R. In Fig. 5, the lowest curve shows how R increases with  $\Delta V$  for a chemical rocket. A mass ratio of 17 corresponds to the  $\Delta V$  for a Hohmann ascent of 12.8 km/s. The vertical ascent mass ratio is off the scale at 50 for a  $\Delta V$  of 17.74 km/s when only chemical-rocket propulsion is used. The Hohmann ascent is preferred for Chemical-rocket propulsion.

If laser propulsion with a specific impulse of 1000 s is used for the final portion of the direct ascent, the total mass ratio is given by the upper curve. The total mass ratio is the product of the mass ratios for the first and second phases. The mass ratio for the second phase is 6.1 at a  $\Delta V_I$  of zero (laser propulsion only) and decreases to 1 at a  $\Delta V_I$  of 17.74 km/s (chemical-rocket propulsion only). Combining the mass ratio for the phases gives a total mass ratio of 6.1 (6.1  $\times$  1) at a  $\Delta V_{i}$ of 0. At a  $V_1$  of 3.5 km/s, the mass ratio for the first phase is 2.1 (lower curve, Fig. 5), the mass ratio for the final phase is 4.3, and the combined mass ratio is about 9 (upper curve). At a  $\Delta V_I$  of about 8 km/s, the total mass ratio for vertical ascent with some laser propulsion is about the same as the mass ratio for a Hohmann ascent with only chemical-rocket propulsion. There is, therefore, no advantage from the vertical ascent with some laser propulsion if  $\Delta V_I$  is 8 km/s or more. If  $\Delta V_I$  is less than 8 km/s, there is a possible advantage from the vertical ascent with some laser propulsion. In fact, if the laser propulsion is used for the entire exo-atmospheric portion of the flight, the advantage may be significant. From Ref. 3, the  $\Delta V$  required before the vehicle leaves the dense portion of the atmosphere is only about 3.5 km/s. For a lower  $\Delta V_I$ , the drag on the laser collector would need to be considered. At a  $\Delta V_{I}$ of 3.5 km/s, the total mass ratio is only about 9, which is nearly low enough to consider using a single-stage vehicle for Earth-to-GEO transportation.

If the laser propulsion system has a specific impulse of 2000 s, the total mass ratio is given by the middle curve of Fig. 5. At the lower values of  $\Delta V_I$ , the increased specific impulse reduces R by over 50%. At a  $\Delta V_I$  of 3.5, the total mass ratio in only 4.5, and single-stage vehicles could definitely be considered.

In Fig. 5, the point at which the vertical thrust ends is noted at a  $\Delta V_I$  of 12,75 km/s. If chemical-rocket propulsion could be used for all of the vertical thrust, which requires a high thrust level, the laser propulsion system could be quite small. Because the  $\Delta V_I$  of 12.75 km/s is itself almost the same as the  $\Delta V_I$  for an entire Hohmann ascent, the laser propulsion must provide some of the vertical thrust for the concept to be attractive.

Figure 5 was prepared based on a  $\Delta V_I$  of 17.74 km/s. This represents the case with only chemical-rocket propulsion. Changing the specific impulse at some point along the trajectory will change the  $\Delta V$  slightly; changing the thrust

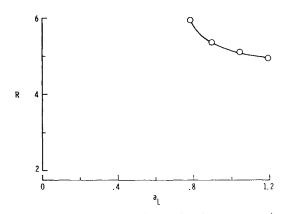


Fig. 6 Effect of laser propulsion acceleration on mass ratio.

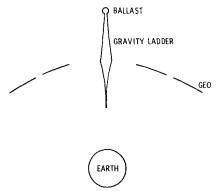


Fig. 7 Illustration of gravity ladder.

history would change the  $\Delta V$  even more. Switching to laser propulsion at a  $\Delta V$  of 3.5 km/s at an altitude of 100 km would require a very high thrust from the laser propulsion if the thrust history shown in Fig. 2 is followed. A more optimal approach would be to use some lower thrust level for the laser propulsion. Figure 6 shows the results of trajectories with different levels of laser thrust intiated at an altitude of 100 km. The laser specific impulse assumed was 2000 s. The mass ratio from Fig. 5 for this case is about 4. As the laser thrust is reduced, the acceleration at the initiation of the laser thrust  $a_I$ falls from 3.0, the value of  $a_m$ . At  $a_L$  values of 1.2-0.8, the mass ratio reaches 5-6 and increases rapidly if  $a_L$  is reduced further. For a laser acceleration of 0.8, the laser thrust required is less than the maximum chemical-rocket thrust by a factor of 4.4. For the initial vehicle gross mass of 1.5 Gg, the laser thrust is 5 MN. The power is 50 GW in the exhaust or about 100 GW required at the laser source. The chemical rockets also produce a power of 50 GW.

#### **Gravity Ladder**

Another advanced transportation concept, the gravity ladder, 5 is compatible with vertical ascent. The gravity ladder, illustrated in Fig. 7, is a long satellite with a central segment in orbit at GEO and two arms. The Earth arm extends towards Earth from the central segment and has less than orbital velocity; it tends to pull the satellite toward Earth. The space arm extends away from Earth from the central segment and has greater than orbital velocity; it tends to pull the satellite away from Earth. In order to balance the satellite without making the space arm extremely long, a ballast is placed at the end of the space arm.

The gravity ladder reduces the requirements on vehicles for space transportation. A vehicle that transports payloads to GEO can dock at the bottom of the gravity ladder, which requires less altitude and velocity than going all the way to GEO. The payload can then be transported up the gravity ladder by an elevator. The ultimate gravity ladder would

reach the Earth's surface, and no vehicle would be required. For transportation into deep space, an elevator could take a vehicle to the ballast end of the gravity ladder. When the vehicle is released there, it has more than circular velocity and can move deeper into space without propulsion. The gravity ladder would need an energy source to transfer payloads along its length and a propulsion system for stationkeeping. The propulsion system could use low-thrust devices and would be much more efficient than vehicle propulsion systems.

The results of an analysis of vertical ascent with a gravity ladder are shown in Fig. 8. The vehicle mass ratio R is shown as a function of the location of the bottom of the gravity ladder. The parameter  $r_0/r_G$  is the radius from the Earth's center to the bottom of the gravity ladder divided by the radius to GEO. The vehicle mass ratio approaches one as the bottom of the gravity ladder approaches Earth's surface. A significant reduction in the vehicle mass ratio requires  $r_0/r_G$  to be several tenths less than one. Therefore, the gravity ladder must be very long and massive to be of much help. For  $r_0/r_G$  of 0.6, the Earth arm is 17,000 km long.

An analysis of the gravity ladder structural requirements and mass is given in the Appendix. The area distribution is exponential, with the maximum area at GEO. The mass calculation required a numerical integration of the area from the bottom of the ladder to GEO to give the dimensionless parameter I. The mass of the Earth arm is then given by  $m = \rho A_0 r_G I$  where  $\rho$  is the material density and  $A_0$  the area at the bottom of the gravity ladder.

The results of the integration are given in Fig. 9 for several values of  $\sigma/\rho$ , where  $\sigma$  is the material strength. For existing fiberglass filaments,  $\sigma/\rho = 2.2 \text{ km}^2/\text{s}^2$ . Graphite whiskers

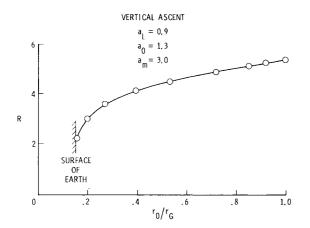


Fig. 8 Effect of gravity ladder on mass ratio.

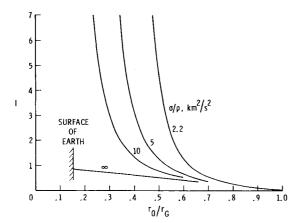


Fig. 9 Results of gravity ladder integration for mass calculation.

may have  $\sigma/\rho$  approaching 10 km²/s². The results indicate that the gravity ladder mass can be large if  $r_0/r_G$  is chosen too low for available materials. A reasonable range for  $r_0/r_G$  for foreseeable material technology is 0.4-0.7. Selecting  $r_0/r_G$  of 0.6 and  $\sigma/\rho$  of 2.2 km²/s² yields I=1.23. With  $\sigma$  of 5500 MN/m² for fiberglass, an elevator and payload mass of 1000 kg can be held by the gravity ladder if  $A_0$  is 1.8 mm². With  $\rho$  of 2500 kg/m³ for fiberglass, the Earth arm mass is then 240 Mg.

A more realistic estimate of the mass of the gravity ladder would need to include a provision for the laser system. Part of it would probably be at the bottom of the gravity ladder. The space arm of the gravity ladder and ballast would probably equal the Earth arm in mass. A total mass of several thousand megagrams might be required.

An evaluation of the feasibility of a gravity ladder is beyond the scope of the effort reported herein, particularly if the dynamics of the gravity ladder are considered. The present results may encourage a more complete evaluation of gravity ladders.

#### **Airbreathing First Stage**

Airbreathing propulsion can provide a higher specific impulse than chemical rockets, but the engine mass is also higher. Earth-to-orbit vehicles with airbreathing first stages have some attractive features. Horizontal takeoff systems keep the engine total mass low by using a low initial acceleration, but the wings must be large. Vertical takeoff with airbreathing engines<sup>6</sup> has been shown to have some benefit, but the return of the first stage appears to be a difficult maneuver for nonvertical Earth-to-orbit trajectories. With vertical ascent, an airbreathing first stage could be used to lift the rest of the vehicle to about 15 km with subsonic velocity using existing turbofan engines. The engines could be arranged in a ring around the vehicle, and recovery could be accomplished by falling back down the vertical path using thrust for braking.

The effect of an airbreathing first stage was estimated by changing the initial altitude to 15 km for the rocket portion of the vehicle. The velocity was assumed to be zero at staging. Unfortunately, the results shown in Fig. 10 indicate the mass ratio for the rest of the flight is not reduced significantly. An airbreathing first stage might allow reduction of the initial rocket acceleration, might reduce launch noise, and might increase safety.

## **Combined Chemical and Laser Propulsion**

One of the undesirable characteristics of the results shown previously is the high thrust level required with laser propulsion. In an attempt to reduce the laser thrust, an investigation was conducted using combined laser and chemical

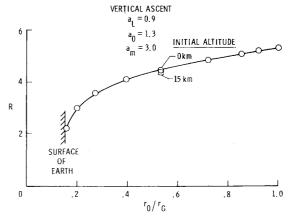


Fig. 10 Effect of initial altitude on mass ratio.

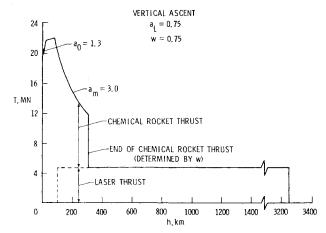


Fig. 11 Thrust history with combined chemical and laser propulsion.

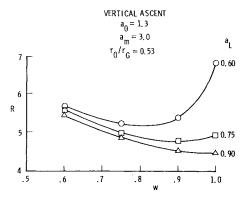


Fig. 12 Effect of combined thrust on mass ratio.

rocket thrust, as shown in Fig. 11. At an altitude of 100 km, the laser propulsion is initiated as before, but the chemical rocket is not turned off. The thrust of the chemical rocket is adjusted to maintain an acceleration  $a_m$ . When the mass reaches some fraction w of the mass at the beginning of the combined burn, the chemical rocket is turned off.

As shown previously for a trajectory with only chemical rockets, at some point the thrust drops and only provides tangential acceleration. In Fig. 11, this drop in thrust is at an altitude of 3254 km. In Fig. 2, the thrust drop occurs at only 1662 km. The difference occurs because the lower acceleration with laser propulsion results in a greater altitude before the velocity is sufficient to allow coasting to the destination altitude.

The results with combined thrust are given in Fig. 12. Without combined thrust (w=1.0), the mass ratio R increases rapidly with reductions in laser acceleration  $a_L$  from 0.90 to 0.75 and 0.60. This trend is the same as that shown in Fig. 6, except that in this case the use of a gravity ladder is included. At high laser thrust levels, such as for  $a_L=0.90$ , the mass ratio R increases with decreased w, which indicates that the specific impulse degradation of combined thrust is more important than the effect of acceleration on gravity losses. At lower levels of  $a_L$ , combined thrust is quite beneficial. At  $a_L=0.60$ , combined thrust can reduce the mass ratio by 23%. At  $a_L=0.75$ , the reduction is 3%. An additional benefit of combined thrust is that the propellant used contains more dense oxygen, so the propellant bulk density is increased, which reduces the vehicle size and inert mass.

#### **Composite Chemical and Laser Propulsion**

Another option for improving the performance of chemical and laser propulsion is shown in Fig. 13. Rather than operate

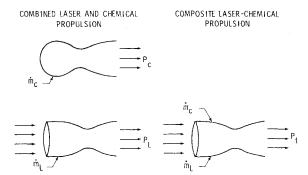


Fig. 13 Illustration of combined and composition propulsion.

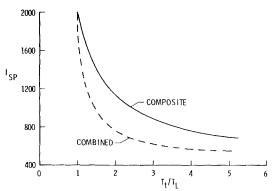


Fig. 14 Effect of composite propulsion on specific impulse.

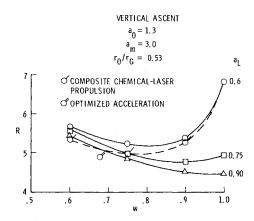


Fig. 15 Effect of composite propulsion on mass ratio.

the two engines separately and in parallel, all of the propellant could be injected into the laser heating chamber. The laser engine is represented by a lens (to focus the incoming laser beam), the heating chamber, and the nozzle. By injecting the chemical propellants into the laser heating chamber, the power of the chemical engine and the power of the laser engine are added, and the total power acts on the total propellant flow. The chamber temperature and the specific impulse are less than for the laser engine but more than for the combined engines, as shown in Fig. 14.

The analysis used to estimate the specific impulses shown in Fig. 14 was based on a simple assumption of constant power. The exhaust velocity for separate chemical and laser exhausts was assumed to be the specific impulse times the reference acceleration of gravity. The power was calculated as one half the mass flow times the velocity squared. For composite propulsion, the power of the two streams was added to get the composite power, and the mass flows were added to get the composite mass flow. The exhaust velocity was calculated as

the square root of twice the power divided by the mass flow. The specific impulse was assumed to be the velocity divided by the reference acceleration of gravity.

Vehicle results with composite propulsion are given in Fig. 15. A noticeable reduction in the mass ratio is shown. The results in Fig. 15 were calculated following the thrusting strategy of Fig. 11, which is to accelerate at  $a_m$  during the periods of composite thrust. For combined propulsion, this strategy is usually best. With composite chemical and laser propulsion, reducing the acceleration when the composite thrust begins would increase the composite specific impulse. One point is shown in Fig. 15 with the acceleration history optimized.

#### Conclusion

This investigation of vertical ascent from Earth to geosynchronous orbit has indicated the following conclusions:

- 1) It is reasonably possible to ascend vertically to geosynchronous orbit without reaching speeds in the atmosphere that would create significant heating.
- 2) The gravity losses with vertical ascent are so great that the concept is not of interest with only chemical-rocket propulsion.
- 3) Vertical ascent is compatible with laser propulsion. If laser propulsion is used for the entire exo-atmospheric portion of the flight, the resulting mass ratio is such that single-stage vehicles might be possible from Earth to geosynchronous orbit.
- 4) Vertical ascent is compatible with the gravity ladder concept. A long satellite in geosynchronous orbit can reduce the propulsion requirements for vertical ascent. Existing or foreseeable materials can be used for a gravity ladder extending approximately half the distance to Earth, but a gravity ladder extending to Earth is not reasonable with foreseeable materials technology.
- 5) Combined chemical and laser thrust is useful with low values of laser thrust, and composite propulsion is even better. The required laser thrust levels are still higher than desired.

### Appendix

The forces that act on an incremental segment of gravity ladder of length dr lead to

$$t + F_F = t + \mathrm{d}t + F_S \tag{A1}$$

where t is the tension below dr and the change in tension, dt, is

$$dt = \sigma dA \tag{A2}$$

where dA is the change in area and  $\sigma$  is the stress. The body force toward Earth is gravitational and is given by

$$F_E = GMdm/r^2 \tag{A3}$$

The gravitational constant is  $G = 66.73 \times 10^{-12} \text{ Nm}^2/\text{kg}^2$ . The mass of Earth is  $M = 5.975 \times 10^{24} \text{ kg}$ . The radius r is measured from the center of Earth. The incremental mass dm is given by

$$dm = \rho A dr \tag{A4}$$

where  $\rho$  is material density. The body force toward space is centrifugal and is given by

$$F_{\rm S} = r\omega^2 \, \mathrm{d}m \tag{A5}$$

The rotation of Earth,  $\omega$ , is once per day or  $72.72 \times 10^{-6}$  rad/s. Combining the above definitions with Eq. (A1) and cancelling t gives

$$GM\rho A dr/r^2 = \sigma dA + \rho A \omega^2 r dr \tag{A6}$$

Dividing by  $\rho A$ , integrating, and evaluating the constant of integration at the bottom of the gravity ladder, noted by ( )<sub>0</sub>, gives

$$-GM\left(\frac{l}{r_0} - \frac{l}{r}\right) - \frac{l}{2}\omega^2(r^2 - r_\theta^2) = \frac{\sigma}{\rho} \ln\left(\frac{A}{A_\theta}\right)$$
 (A7)

This can be solved for the area to give

$$\frac{A}{A_0} = \exp\left\{\frac{1}{\sigma/\rho} \left[GM\left(\frac{l}{r_0} - \frac{l}{r}\right) - \frac{1}{2}\omega^2 \left(r^2 - r_0^2\right)\right]\right\}$$
(A8)

Equation (A4) can be integrated to find the mass of the Earth arm, using the geosynchronous radius,  $r_G$ , 42.2  $2 \times 10^6$  m, to nondimensionalize  $r_s$ 

$$M = \rho A_0 r_C I \tag{A9}$$

where

$$I = \frac{1}{r_{\theta}/r_{G}} \left( \frac{A}{A_{\theta}} \right) d\left( \frac{r}{r_{G}} \right)$$

and Eq. (A8) must be substituted for  $(A/A_{\theta})$ .

#### References

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<sup>2</sup>Brauer, G.L., Cornick, D.E., and Stevenson, R., "Capabilities

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